Uitgewerkte versie

Exam Regeltechniek TBKRT05E

Tuesday 6 November 2012, 9:00 - 12:00 uur

Name: Student ID:

- This exam consists of 14 pages with 7 open questions. Check if you have all pages.
- The answers to the questions (including motivation for ALL answers) have to be placed in the answer boxes.
- Please put your name and student number at all pages. The exercises will be collected separately.
- If you like, you can add additional paper, which need to include your name and student number. Please provide SEPARATE papers for separate exercises.
- The use of book, reader and course material is allowed, under the condition that everything is tight into one bundle. E-readers, PDA's, tablets, etc., are NOT allowed.
- To obtain a grade, the practical has to have been finalized in 2011 or 2012.
- Good luck!

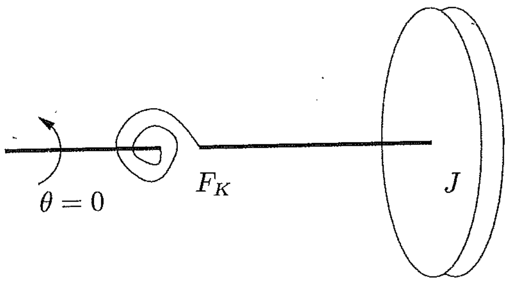
Exam TBKRT05E, 6 November 2012	Uitwerking
Name:	Student ID:

Excercise 1 (18 points)

Given is a rotating mass connected to a rotating spring (i.e., instead of the displacement, the spring works on the angular displacement). The mass has an inertia J and the rotational spring is nonlinear with spring force

$$F_K = K\theta + \frac{K}{4}\theta^3.$$

The left side of the spring is connected, and thus has angle $\theta = 0$. An external torque τ is applied to the rotating mass.



a). Provide the Lagrangian and the Euler-Lagrange equations. Motivate your answer!

$$\mathcal{L}(\theta,\dot\theta) = \frac{1}{2}J\dot\theta^2 - \frac{1}{2}K\theta^2 - \frac{K\theta^4}{16}$$

$$\frac{\partial \mathcal{L}}{\partial \dot\theta} = J\dot\theta \qquad \frac{\partial \mathcal{L}}{\partial \theta} = -K\theta - \frac{1}{4}K\theta^3 \qquad \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot\theta} - \frac{\partial \mathcal{L}}{\partial \theta} = \tau \Rightarrow$$

5

$$J\ddot\theta + K\theta + \frac{1}{4}K\theta^3 = \tau$$

b). Now assume that the rotating mass is damped by a linear friction force (linear with the angular velocity) with constant b. Provide the Rayleigh dissipation function and the corresponding Euler-Lagrange equations. Motivate your answer!

$$R(\dot\theta) = \frac{1}{2}b\dot\theta^2 \qquad \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot\theta} - \frac{\partial \mathcal{L}}{\partial \theta} = \tau - \frac{\partial R}{\partial \dot\theta} \quad \text{Hence}$$

$$\frac{\partial R}{\partial \dot\theta} = b\dot\theta$$

4

$$J\ddot\theta + b\dot\theta + K\theta + \frac{1}{4}K\theta^3 = \tau$$

Exam TBKRT05E, 6 November 2012

Name: Student ID:

c). Determine a state space model from the Euler-Lagrange equations. Motivate your answer!

For example choose $x_1 = \theta$, $x_2 = \dot{\theta}$, then and $u = \tau$

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\frac{b}{J} x_2 - \frac{K}{J} x_1 - \frac{K}{4J} x_1^3 + \frac{1}{J} u \end{cases}$$

$\hookrightarrow$ follows from the EL equation of b)

4

d). Now take as spring force a linear rotational spring, i.e., $F_K = K\theta$. Take $J = 2$, $K = 3$ en $b = 5$ and take as input the torque τ and as output the angular position of the rotating mass. Determine the transfer function. Motivate your answer!

linear spring force, hence state space model

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\frac{b}{J} x_2 - \frac{K}{J} x_1 + \frac{1}{J} u \end{cases} \qquad A = \begin{pmatrix} 0 & 1 \\ -\frac{K}{J} & -\frac{b}{J} \end{pmatrix} \quad B = \begin{pmatrix} 0 \\ \frac{1}{J} \end{pmatrix}$$

$$y = x_1 \qquad C = \begin{pmatrix} 1 & 0 \end{pmatrix} \qquad D = 0$$

5 With $J = 2$, $K = 3$, $b = 5$ $\quad A = \begin{pmatrix} 0 & 1 \\ -\frac{3}{2} & -\frac{5}{2} \end{pmatrix}$, $B = \begin{pmatrix} 0 \\ \frac{1}{2} \end{pmatrix}$, $C = \begin{pmatrix} 1 & 0 \end{pmatrix}$

$$(sI - A)^{-1} = \begin{pmatrix} s & -1 \\ \frac{3}{2} & s + \frac{5}{2} \end{pmatrix}^{-1} = \frac{1}{s(s+\frac{5}{2}) + \frac{3}{2}} \begin{pmatrix} s + \frac{5}{2} & 1 \\ -\frac{3}{2} & s \end{pmatrix}$$

$$C(sI - A)^{-1} = \frac{1}{s^2 + \frac{5}{2}s + \frac{3}{2}} \begin{pmatrix} s + \frac{5}{2} & 1 \end{pmatrix}$$

$$C(sI - A)^{-1}B = \frac{\frac{1}{2}}{s^2 + \frac{5}{2}s + \frac{3}{2}} = \frac{1}{2s^2 + 5s + 3}$$

Exam TBKRT05E, 6 November 2012 Uitwerkingen .

Name: Student ID:

Excercise 2 (18 punten)

Consider a model of the DC motor with neglectable inductance and nonlinear damping (drag). If the inductance is very small:

$$J_m \ddot{\theta}_m + b\dot{\theta}_m^2 = \frac{K_m}{R_a} v_a - \frac{K_m K_e}{R_a} \dot{\theta}_m$$

with motor voltage v_a, motor speed $\omega_m = \dot{\theta}_m$, J_m is the inertia, K_m the motor constant for the mechanical connection with the electrical part, K_e the motor constant for the electrical connection with the mechanical part, R_a the electrical resistance, and b a constant corresponding to the nonlinear damping.

a). Take as state $x = \omega_m$, input $u = v_a$, and output $y = \omega_m$. Write the state space equations, and determine the equilibrium point(s) for $u = 1$. Motivate your answer!

$x = \dot{\theta}_m$

$$\begin{cases} \dot{x} = -\frac{b}{J_m} x^2 - \frac{K_m K_e}{J_m R_a} x + \frac{K_m}{J_m R_a} u \\ y = x \end{cases}$$

7

$\dot{x} = 0$ for $u = 1$ $\Leftrightarrow$ $b x^2 + \frac{K_m K_e}{R_a} x = \frac{K_m}{R_a}$

hence, two eq. points.

$\Leftrightarrow$ $x = -\frac{K_m K_e}{2 R_a b} \pm \frac{1}{2b} \sqrt{\frac{K_m^2 K_e^2}{R_a^2} + \frac{K_m 4b}{R_a}}$

b). Linearize the system around the point $u = 0, x = 0$. Is this linear system stable, asymptotically stable or instable? Motivate your answer!

$u = 0$, $x = 0$. If $u = 0$, then $x = 0$ eq. point.

$f(x) = -\frac{b}{J_m} x^2 - \frac{K_m K_e}{J_m R_a} x$ $g(x) = B = \left(\frac{K_m}{J_m R_a}\right)$

$h(x) = C = 1$

$D = 0$

7

$$\frac{\partial f}{\partial x}\Big|_{(0,0)} = \left(-\frac{2b}{J_m} x - \frac{K_m K_e}{J_m R_a}\right)\Big|_{(0)} = -\frac{K_m K_e}{J_m R_a} = A$$

If $K_m, K_e, J_m, R_a > 0$, then $A < 0$, hence as. stable.

Please turn over

~~$\Delta\dot{x} = A\Delta x + B\Delta u$~~

~~$y = C\Delta x$~~

Exam TBKRT05E, 6 November 2012

Name: Student ID:

$$\Delta\dot{x} = A\Delta x + B\Delta u$$
$$\Delta y = C\Delta x$$

$\Delta x = x - x^* = x$
$\Delta u = u - u^* = u$

c). Linearize the system about the point $u = 1, x = -1$ and $u = 1$, $x = 0.5$. Are these linear systems stable, asyptotically stable or instable? Motivate your answer!

The B and C matrices are the same
(1x1 matrices, or in other words scalars

$$B = \frac{K_m}{J_m R_a} \qquad C = 1$$

$\Delta\dot{x}_1 = A_1\Delta x_1 + B\Delta u$
$\Delta y_1 = C\Delta x_1$
$\Delta x = x - x^* = x+1$
$\Delta u = u - 1$

$$A_1 = \frac{\partial f}{\partial x}\Big|_{x=-1} = \frac{2b}{J_m} - \frac{K_m K_e}{J_m R_a}$$

$\rightarrow$ depends on values for

$$A_2 = \frac{\partial f}{\partial x}\Big|_{x=0.5} = -\frac{b}{J_m} - \frac{K_m K_e}{J_m R_a} < 0$$

$\Delta\dot{x}_2 = A_2\Delta x_2 + B\Delta u$
$\Delta y_2 = C\Delta x_2$
$\Delta x_2 = x - 0.5$ $\quad \Delta u = u - 1$

if $J_m, K_m, K_e, R_a > 0$
and hence as. stable

if $\dfrac{K_m K_e}{R_a} > 2b$ for $J_m, K_m, K_e, R_a > 0$ then as. stable.

if $\dfrac{K_m K_e}{R_a} = 2b$ then stable

if $\dfrac{K_m K_e}{R_a} < 2b$ then instable.

4

Exam TBKRT05E, 6 November 2012	Uitwerkingen
Name:	Student ID:

Excercise 3 (18 punten)

Consider the following continuous time state space system with input $u(t)$, and a constant a.

$$\dot{x}(t) = \begin{pmatrix} 1 & 5 \\ a & -5 \end{pmatrix} x(t) + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u(t)$$

a). For which a is this system stable, asymptotically stable, unstable? Motivate your answer!

$\det(\lambda I - A) = (\lambda-1)(\lambda+5) - 5a = 0$

$\Leftrightarrow \quad \lambda^2 + 4\lambda - 5 - 5a = 0$

$\Leftrightarrow \quad \lambda = -2 \pm \frac{1}{2}\sqrt{16+20+20a}$

$Re\{\lambda\} < 0$ if $20+20a < 0 \Leftrightarrow a < -1 \rightarrow$ as stable

$Re\{\lambda\} = 0$ if $a = -1 \rightarrow$ stable

$Re\{\lambda\} > 0$ if $a > -1 \rightarrow$ unstable

5

b). Determine for which a the system is reachable. Motivate your answer!

$$W_r = [B \quad AB] = \begin{bmatrix} 1 & 1 \\ 0 & a \end{bmatrix}$$

4

$\det W_r \neq 0$ if $a \neq 0$. Hence reachable if $a \neq 0$, not reachable if $a = 0$

Please turn over

c). Determine the state feedback $u = -Kx + k_r r$ for $a = 1$ such that the closed loop system is stable for reference signal r, and such that the closed-loop poles are given by -1 and -2. Motivate your answer!

You can follow Theorem 6.3 from the book:

$$K = [p_1 - a_1 \quad \cdots \quad p_n - a_n]\, \tilde{W}_r W_r^{-1}$$

Char. polynomial of A matrix: $\lambda^2 + 4\lambda - 10$

$\Rightarrow a_1 = 4, \quad a_2 = -10$

Char. polynomial of closed loop: $(\lambda+2)(\lambda+1) = \lambda^2 + 3\lambda + 2$

$\Rightarrow p_1 = 3, \quad p_2 = 2$

6 $\tilde{W}_r = \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & -4 \\ 0 & 1 \end{pmatrix}$

W_r from exercise b): $W_r = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

$\Rightarrow K = [-1 \quad 12] \begin{pmatrix} 1 & -4 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} = [-1 \quad 12] \begin{pmatrix} 1 & -5 \\ 0 & 1 \end{pmatrix} = [-1 \quad 17]$

In addition $k_r = -1 / C(A-BK)^{-1}B =$ with output x_1, $C = [1 \quad 0]$

$-1 / [1 \quad 0] \begin{pmatrix} 2 & -12 \\ 1 & -5 \end{pmatrix}^{-1} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = -\dfrac{1}{[1 \quad 0] \begin{bmatrix} -2\frac{1}{2} & 6 \\ -\frac{1}{2} & 1 \end{bmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}} = -\dfrac{1}{2\frac{1}{2}} = -\dfrac{2}{5}$

d). Is the closed loop system underdamped, overdamped or critically damped? Motivate your answer!

Char. polynomial $\lambda^2 + 3\lambda + 2$

$\Leftrightarrow \omega_n^2 = 2 \Leftrightarrow \omega_n = \sqrt{2}$. $\quad 2\zeta\omega_n = 3 \Leftrightarrow \zeta = \dfrac{3}{2\sqrt{2}}$

3 $\approx 1{,}0671$

hence the closed loop system is overdamped!

Exercise 4 (8 points)

Given a transfer function of the closed loop system as

$$T(s) = \frac{b_0 s + b_1}{s^2 + a_1 s + a_2}.$$

The controller was designed such that the closed loop system has a DC gain (the gain for $\omega = 0$) equal to 1 and a rise time $t_r < 0.2sec$ (where the rise time t_r is assumed to be equal to the rise time of the system $\frac{1}{s^2+a_1 s+a_2}$). What can you conclude for the system parameters b_0, b_1, a_1, a_2 so that T fulfills these specifications? Motivate your answer! (hint: not all parameters are restricted by these specifications).

Recall that $t_r \approx \frac{1.8}{\omega_n}$ $\quad \omega_n = \sqrt{a_2}$

Hence $t_r = \frac{1.8}{\sqrt{a_2}} < 0.2 \Rightarrow a_2 > 81$ 4 punten

DC gain $T(0) = \frac{b_1}{a_2} = 1 \Leftrightarrow b_1 = a_2$

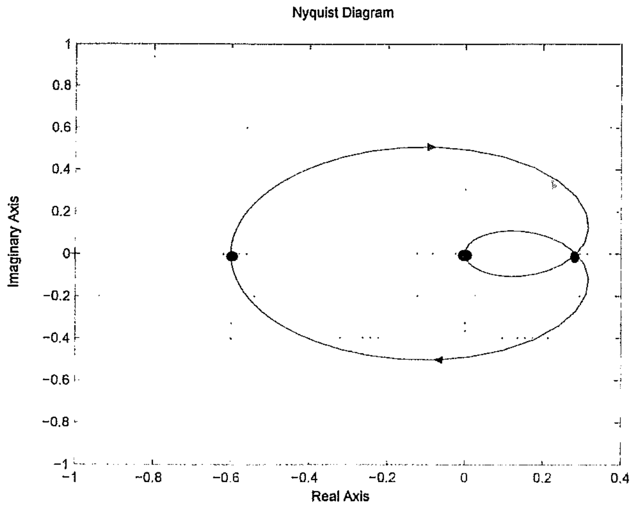
hence $b_1 > 81$ 4 punten.

Please turn over

Exam TBKRT05E, 6 November 2012	
Name:	Student ID:

Exercise 5 (15 points)

Consider the Nyquist plot of a transfer function $G(s)$ of a stable system as follows:



a). For which values of K is the standard closed loop system for loop transfer $L(s) = KG(s)$ stable? Motivate your answer!

7

$GM = \frac{1}{0.6}$. Hence for $K > \frac{1}{0.6} \approx 1,7$ the system becomes unstable since it then will encircle -1. Stable for $K < 1.7$, 3pt

4

1pt

b). Assume that $K = 5$. Provide N, P and Z from the Nyquist criterium. Is the closed loop system stable? Motivate your answer!

8

See a), $K = 5 > 1,7$, hence unstable

G is stable, hence $P = 0$

Number of encirclements of -1: $N = 1$

Hence $Z = N + P = 1$, the closed loop system has 1 unstable pole.

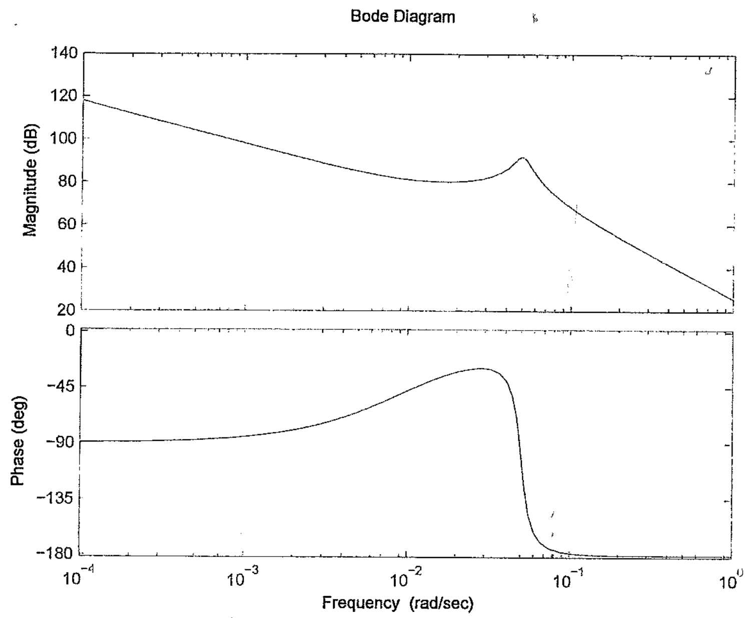
N = 2pt
P 2
Z 2
concl.
Z≠0 unstable 3pt 1pt
unstable pole

Exercise 6 (15 points)

For the design of a controller for an airplane the following model is being used:

$$G(s) = \frac{80(100s+1)}{s(400s^2+4s+1)}.$$

The Bode plot of the system is given by



The system can be treated as a second order system. Design a lead controller

$$D(s) = K\frac{Ts+1}{\alpha Ts+1}$$

such that

- The cross-over frequency $\omega_c = 0.1$.
- The phase margin $PM \geq 50$.

Hint: first determine α, then T and then K. Motivate your answer!

A $\omega_c = 0.1$, 50° phase lead needs to be added. Relation between phase lead and α is given

Hence $\alpha = \dfrac{1 - \sin 50^\circ}{1 + \sin 50^\circ} \approx \dfrac{1}{8} \approx 0{,}13$

Please turn over

1,71 voor 50 rad

Furthermore, for a lead we have maximum phase addition at

$$\omega = \frac{1}{T\sqrt{\alpha}},$$

which we want to be the new cross over frequency. Hence

$$\omega_c = 0.1 = \frac{1}{T\sqrt{\frac{1}{8}}} \Rightarrow T \approx 28{,}3$$

Now the appropriate cross over frequency is determined by adjusting K, i.e., for $\omega = 0.1$, we have $|G(0.1j)| = 2{,}663 * 10^3$ (either calculated or from plot)

and $|D(0.1j)| = K \cdot 2{,}83$. To have $\omega_c = 0.1$, it needs to be that

$|G(0.1j)|\,|D(0.1j)| = 1 \iff K \approx 1{,}33 \cdot 10^{-4}$

	volledig	geen motivatie	alleen correcte formule
α	5pt	3	1
T	5pt	3	1
K	5pt	3	1

Exam TBKRT05E, 6 November 2012

Name: Student ID:

Exercise 7 (12 points of which 4 are bonus)

Consider for example the simple model of a DC motor given by

$$G(s) = \frac{1}{s(s+1)}$$

Explain when you would advice to design a lead compensator, and when to design a lag compensator. Provide a short and simple manual for engineers to help them decide what to design, and based upon which principles.

See slides lecture 12 of book. For example:

* A lead network increases the ratio $\frac{\omega_c}{DG(0)}$ hence if ω_c is constant the low frequency gain is decreasing, or if the low frequency gain is constant, ω_c increases

* A lag network decreases the ratio $\frac{\omega_c}{DG(0)}$, hence precisely opposite to the above.

In general, ω_c large is not so good. due to (amplification of high frequency noise!) ~~Hence~~ This is linked to bandwidth and other consideration (often the higher frequencies correspond to noise and disturbances, and then we want the magnitude to be small).

END EXAM